THE MATHEMATICAL PATTERN OF THE HOUR INDEX AT THE THREE-PHASE TRANSFORMERS. CONSIDERATIONS ON THE DEPHASING MATRIX

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Abstract. *The paper presents the carried out researches' results within Department of Scientific Research for domain of Machines, Devices and Electrical Controls – EMAD of "Stefan cel Mare" University of Suceava concerning errors identification of the connections drafts and imperfections diagnosis at three – phased transformers. Authors' contributions are based on the existent connection between the real configuration of the draft of connections and of mathematical sample of the index of hours, expressed under the form of code matrix. There are analyzed mathematical expressions of some errors, as circular permutation of terminals connections, reversing beginning and ending at phase windings, reversing the connections for terminals, changing the connections from N into Z at triangle connections, zig – zag, etc. There are also analyzed the using possibilities for mathematical expressions obtained through identification of mentioned errors, with computer provided help. The end of paper is intended for presenting the resulted conclusions after the experimental study.*
Keywords: *index of hours, mathematical sample, errors, three – phased transformers.*

Preliminary considerations

For the identification of an hour index after a modification of a transformer, usually, is used a vectorial construction (that claims to know a connections' scheme) or an experimental method. The authors are proposing and analysing another solution, that utilises the mathematical pattern of the hour index and its relation with the configuration of the connection schemes, belonging to the modified transformer.
In [1] is presented a mathematical model of the hour index, expressed by a code equation having the following form:

$$[G_k] = \begin{bmatrix} \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{21} & \eta_{22} & \eta_{23} \\ \eta_{31} & \eta_{32} & \eta_{33} \end{bmatrix} \tag{1}$$

In the same work is proposed the decomposition of this matrix, in three elementary matrixes, as it follows:

$$[G_k] = [E_a] + [E_b] + [E_c] \tag{2}$$

in which:

$$[E_a] = \begin{bmatrix} \eta_{11} & 0 & 0 \\ 0 & \eta_{22} & 0 \\ 0 & 0 & \eta_{33} \end{bmatrix} \tag{3'}$$

$$[E_b] = \begin{bmatrix} 0 & \eta_{12} & 0 \\ 0 & 0 & \eta_{23} \\ \eta_{31} & 0 & 0 \end{bmatrix} \tag{3''}$$

$$[E_c] = \begin{bmatrix} 0 & 0 & \eta_{13} \\ \eta_{21} & 0 & 0 \\ 0 & \eta_{32} & 0 \end{bmatrix} \tag{3'''}$$

This decomposition is preferred because the matrixes [E_a], [E_b], [E_c] display the general properties of the code matrix:
- the elements placed on the main diagonal and the ones placed on directions parallel to the main diagonal are always even;
- the elements of a raw or of a column, always result from the previous raw or column, by a circular permutation.

The matrix equation (2) suggests the generalised scheme at fig.1, used to model the code matrix for the hour index.

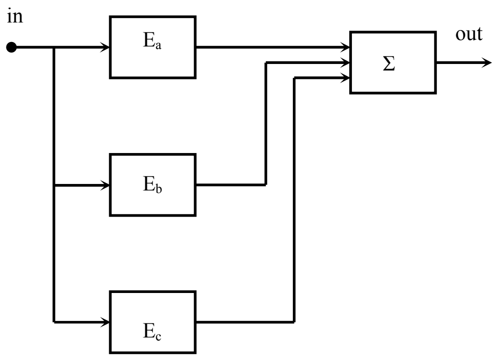

Fig. 1 The generalised scheme for modelling the code matrix. E_a, E_b, E_c - schemes for modelling the elementary matrixes [E_a], [E_b], respectively [E_c]; Σ – the summing unity. Reproduction from [1]

The elements η_{ij} may have the values of „-1”, „0” or „1”.The authors took into consideration only the cases in which η_{ij} is unitary, and the following matrixes have been obtained, as it follows:

$$[E_a]=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=[E_{100}] \tag{4'}$$

$$[E_b]=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}=[E_{10}] \tag{4''}$$

$$[E_c]=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}=[E_{1}] \tag{4'''}$$

Considerations on the dephasing matrix

After a modification made to one of the transformers, the initial matrix is modified.
These modifications are observing an equation, that the authors are calling the transformations equation:

$$[G_k]=(-1)^m\cdot[D]^n\cdot[G_i] \tag{5}$$

in which:
$[G_k]$ - the matrix resulted by modifications;
$[G_i]$ - the initial matrix;
m - constant that may have two values: “1” or “2”;
n – constant that may have three values: “1”, “2” or “3”;
$[D]$ - the dephasing matrix.

The dephasing matrix has two proprieties:
1) $[D]^1=[E_1]$; $[D]^2=[E_{10}]$; $[D]^3=[E_{100}]$.
2) The elements of a raw or a column are resulting from the previous raw or column, by a clockwise circular permutation.
3) Each raw or column is made of two null elements and by an element equal to the unity.
4) The elements placed on the main diagonal are always null.
5) $[D]_T=[E_{10}]$; $\{[D]_T\}_T=[E_1]$.
6) The circular permutation of the columns in direct sense , leads to $[D]^2$ or to $[E_{10}]$.
7) The circular permutation of the colums in reversed sense, leads to $[D]^3$ or to $[E_{100}]$.
8) The circular permutation of the rows in direct sense, leads to $[D]^3$ or to $[E_{100}]$.
9) The circular permutation of the rows in reversed sense, leads to $[D]^2$ or to $[E_{10}]$.

Starting with the main changes that may occur in a transformer and with the transformations equation (5), the authors have succeeded in expressing analytically the following changes:
-circular permutations in direct sense of the terminals’ couplings in the primary:

$$[G_k]=(-1)^2\cdot[D]^1\cdot[G_i] \tag{6}$$

In this case, m=2 and n=1.
-circular permutations in inverse sense of the terminals' couplings in the primary:

$$[G_k]=(-1)^2\cdot[D]^2\cdot[G_i] \tag{7}$$

In this case, m=2 and n=2.
-circular permutations in direct sense of the terminals' couplings in the secondary:

$$[G_k]=(-1)^2\cdot[D]^2\cdot[G_i] \tag{8}$$

In this case, m=2 and n=2.
-circular permutations in inverse sense of the terminals' couplings in the secondary:

$$[G_k]=(-1)^2\cdot[D]^1\cdot[G_i] \tag{9}$$

In this case, m=2 and n=1.
-changing the alimentation from the high voltage wrap to the low one:

- for the odd groups:

$$[G_k]=(-1)^1\cdot[D]^n\cdot[G_i] \tag{10}$$

- for the even groups:

$$[G_k]=(-1)^2\cdot[D]^n\cdot[G_i] \tag{11}$$

in which:
m= 1 for the odd groups and m=2 for the even groups;
n=1 for the groups 1, 4, 7, and 10;
n=2 for the groups 2, 5, 8, and 11;
n=3 for the groups 3, 6, 9, and 12.
-modifying the initial scheme through a type 3I variant (3I- reversing the coiling sense, reversing the terminals' couplings, reversing the terminals' notations):

$$[G_k]=(-1)^1\cdot[D]^3\cdot[G_i] \tag{12}$$

In this case, m=1 and n=3.
-modifying the couplings between the phase wrappings at the connections d (triangle) and Z (zig-zag):

1. modifying the couplings from N to Z at the high voltage wrappings:

$$[G_k]=(-1)^1\cdot[D]^1\cdot[G_i] \tag{13}$$

In this case, m=1 and n=1.

2. modifying the couplings from Z to N at the high voltage wrappings:

$$[G_k]=(-1)^1\cdot[D]^2\cdot[G_i] \tag{14}$$

In this case, m=1 and n=2.

3. modifying the couplings from N to Z at the low voltage wrappings:

$$[G_k]=(-1)^1\cdot[D]^2\cdot[G_i] \tag{15}$$

In this case, m=1 and n=2.

4. modifying the couplings from Z to N at the low voltage wrappings:

$$[G_k]=(-1)^1\cdot[D]^1\cdot[G_i] \tag{16}$$

In this case, m=1 and n=1.
-reversing the couplings between two terminals in the primary and two in the secondary:

- for the odd groups:

$$[G_k]=(-1)^1\cdot[D]^n\cdot[G_i] \tag{17}$$

- for the even groups:

$$[G_k]=(-1)^2\cdot[D]^n\cdot[G_i] \tag{18}$$

in which:
m= 1 for the odd groups and m=2 for the even groups.

1. - reversing the couplings between A and B, respectively a and b:

n=1 for the groups 1, 4, 7, and 10;
n=2 for the groups 2, 5, 8, and 11;
n=3 for the groups 3, 6, 9, and 12.
This relations is the same like when we reverse the conections between B and C, respetively b and c, and also, when we reverse the conections between A and C, respetively a and c.

2. reversing the couplings between A and B, respectively a and c:

n=1 for the groups 2, 5, 8, and 11;
n=2 for the groups 3, 6, 9, and 12;
n=3 for the groups 1, 4, 7, and 10.

This relations is the same like when we reverse the conections between B and C, respetively b and a, and also, when we reverse the conections between A and C, respetively b and c.

3. reversing the couplings between A and B, respectively b and c:

n=1 for the groups 3, 6, 9, and 12;
n=2 for the groups 1, 4, 7, and 10;
n=3 for the groups 2, 5, 8, and 11.

This relations is the same like when we reverse the conections between B and C, respetively c and a, and also, when we reverse the conections between A and C, respetively b and a.

Conclusions

Establishing the analitical expressions that define the various modifications wich may appear in the connections scheme of a transformer opens the possibility to establish a diagnosis of the assembling errors in the transformer's scheme.

As we noticed, all these montage errors are observing the same equation, respectively the one denominated by the authors as the transformations equation.

This equation is showing that, consequently to a modification occurred in a transformer, we may obtain the modified matrix only by multiplicating the initial matrix with another one , that we may denominate the dephasing matrix.

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